



## MODELING OF ELECTRIC FIELD AROUND 100 MVA 150/20 kV POWER TRANSFORMATOR USING CHARGE SIMULATION METHOD

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### Abstract

Charge Simulation Method is one of the field theory that can be used as an approach to calculate the electromagnetic distribution on the electrical conductor. This paper discussed electric field modeling around power transformer by using Matlab to find the safety distance. The safe distance threshold of the electric field to human health refers to WHO and SNI was 5 kV/m. The specification of the power transformer was three phases, 150/20 kV, and 100 MVA. The basic concept is to change the distribution charge on the conductor or dielectric polarization charge with a set of discrete fictitious charge. The value of discrete fictitious charge was equivalent to the potential value of the conductor, and became a reference to calculate the electric field around the surface contour of the selected power transformer. The measurement distance was 5 meter on each side of the transformer surface. The results showed that the magnitude of the electric field at the front side was 5541 V/m, exceeding the safety limits.

Keywords: electric field, charge simulation method, discrete charge, power transformer.

### I. INTRODUCTION

The electric field is an area that is still influenced by the electrical properties of a particular charge. An electric field presents in any region where a charged object experiences an electric force. This is a fancy way of saying that the only way we can conclude the present of electric field is by performing a test charge on that spot. In nature, the electric field generated by the natural formation of electrical charges in the atmosphere associated with lightning. The electric field also exists due to the electrical equipment such as generators, transformers, transmission lines, distribution lines, and other electric and electronic equipment. However, the influence of an electric field around high voltage equipment is greater than the effect of electric fields that exist in nature. The existence of an electric field can indirectly cause problems to human health. It is depending on how powerful electric field and magnetic field exposed to the

human body that can cause problems. Population growth and technological change have led to increase in demand for electrical energy in larger quantities. This causes enhancement of electric field pollution in the urban and the work environments [1].

As it is stated before that the electric field can affect human health, it is also shown risk of cancer death in children living near the transmission line of high voltage [2]. In the 1960 and early 1970, the Soviet Union reported health effects experienced by their workers in high voltage switch yard. Within several months the first 500 kV substations operated in the Soviet Union, maintenance workers complained about headaches, sexual potency reduction, and general ill health. The electric field was assumed to be responsible for the health complaints. Personnel working with 500 kV and 750 kV lines were compared to the workers at 110kV and 220 kV substations. Maximum intensities within the 500 kV and 750 kV switchyard were generally between 15 kV/m and 25 kV/m and biological effects were reported above 5 kV/m[3].

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According to the WHO standard in 1990, boundary electric field allowed in the public areas for 24 hours per day is 5 kV/m [4]. The Indonesian government adopted the IRPA and WHO recommendations which then included in the Indonesian National Standards (SNI) concerning the safety limit for the influence of a 50-60 Hz electric field, as shown in Table 1.

There is a lot of high voltage electrical equipment used in electric power systems one of which is the power transformer. Power transformer is a device that is used to convert inbound electricity or voltage to a higher or lower value in order to accommodate the current flow needed for specific purposes. Power transformer are a normal component in the power grids of many nations, making it possible to regulate the transfer of power to residences and commercial buildings without overloading the circuitry in those structures [5]. There are various levels of voltage used in power distribution systems, i.e. Extra High Voltage (500/150 kV), High Voltage (150/70 kV), Medium Voltage (150/20 kV, 70/20 kV), and Low Voltage (20 kV/380 V). Since the area around the power transformer has a large electric field, it is important to determine the safety distance from electric field effect for human health according to WHO and SNI.

This paper discusses the electric field that is produced by power transformer 100MVA, 150/20 kV, 50 Hz, from different sides of a surface with reference to each measurement distance of 5 meters. The modeling of electric field used the Charge Simulation Method (CSM) which applied to the Matlab. The aims is to know the magnitude of electric field generated by influence of power transformer 100 MVA, 150/20 kV, 50 Hz, to a safe distance according to WHO standard.

## II. BASIC THEORY

### A. Electric Field

The electric field can be analyzed as an electrostatic field and magneto static, the electric field generated by AC-current is quasi-static [6].

Table 1.  
The threshold electric field and magnetic field 50-60 Hz

No	Classification	Electric Field (kV/m)
1	Working area: - Working time - Short time	10 30 (0 - 2 hour/day)
2	Public area : - 24 hour/day - Few hour/day	5 10

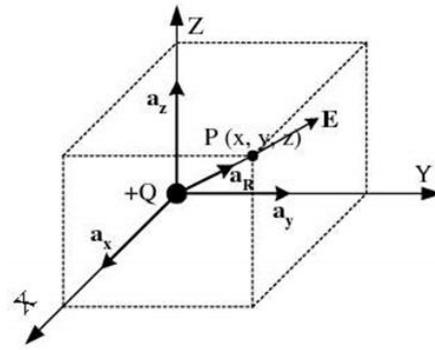


Figure 1. The point Pis The Vector Sum of The Electric field due to charge+Q

It also produced by the voltage from electrical equipment. The strength of an electric field at a given point in space near an electrically charged object is proportional to the amount of charge on the object, and inversely proportional to the distance between the point and the object.

Electric fields strength are usually denoted by the symbol  $E$ , and it is a vector that has both magnitude and direction, as defined below [7]:

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \cdot a_R \quad (1)$$

where

$E$  : Electric field strength (V/m)

$Q$  : Point charge (Coloumb)

$\epsilon_0$  : Permittivity dielectric

$R$  : Distance between Point charge and point P

If described in Cartesian coordinates, the electric field  $E$  of a point charged  $+Q$  and lies in the coordinates of the point  $P(x, y, z)$  seen as vectors as shown in Figure 1.

If there are many charges on the different positions, the fields caused by n point charges can be written as follow [8]:

$$E(r) = \frac{Q_1}{4\pi\epsilon_0 |r-r_1|^2} \cdot a_1 + \frac{Q_2}{4\pi\epsilon_0 |r-r_2|^2} \cdot a_2 + \dots + \frac{Q_n}{4\pi\epsilon_0 |r-r_n|^2} \cdot a_n \quad (2)$$

where

$E(r)$  : electric field strength at point r (V/m)

$Q_n$  : point charge at point n (Coloumb)

$\epsilon_0$  : permittivity dielectric

$r$  : the position of the observation point

$r_n$  : the position of the sources

$a_n$  : unit vector  $n$ .

For homogeneous line charge with charge density per unit length  $\rho_L$ , then equation of the electric field above will be shown in eq. (3):

$$E(\rho) = \frac{Q_L}{2\pi\epsilon_0 \rho_L} \cdot a_\rho \quad (3)$$

where

$E(\rho)$ : electric field strength due to line charge (V/m)

$Q_L$  : line charge (Coloumb)

$\epsilon_0$  : permittivity dielectric

$\rho_L$  : length (m)

**B. Charge Simulation Method (CSM)**

Charge Simulation Method (CSM) is a method that used as an approach to distribution electric field problem induced by the charged conductor CSM considered a practical method for calculating the fields and from its simplicity in representing the equipotential surfaces of the electrodes, its application to unbounded arrangements whose boundaries extend to infinity and its direct determination to the electric field [9]. The magnitudes of fictitious discrete charges are equivalent to the potential value of the conductor and used as a reference to calculate the electric field around the contour of the conductor surface that selected. Once the values and positions of simulation charges are known, the potential and field distribution anywhere in the region can be computed easily [10].

In many practical problems of electrostatic, charge always located near a conductor. An electron released by an electrode and a power transmission line suspended over the earth conductor is an example of a commonly encountered. Let's review the case of a point charge near an infinite plane conductor, as shown in Figure 3, in determining the potential +q with height d using Poisson equation with  $z > 0$ , and boundary condition  $V = 0$  at  $z = 0$  and at infinity [11].

Since there is no conductor, the solution to find the point charge in free space is as follows:

$$V(x, y, z) = \frac{q}{4\pi\epsilon} \left( \frac{1}{\sqrt{x^2+y^2+(z-d)^2}} \right) \quad (4)$$

where

$V$  : Electric potential (V)

$q$  : point charge (Coloumb)

$\epsilon$  : Permittivity dielectric

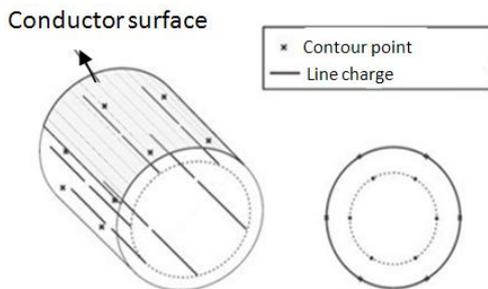


Figure 2. Discretization charges on the rod conductor

Potential function  $V$  on equation (4) accomplishes Poisson's equation for  $z > 0$  with boundary condition  $V = 0$ . However, the potential is not zero on  $z = 0$ . So the solution of case above can be shown in Figure 4 as follow:

The potential is expressed as:

$$V(x, y, z) = \frac{q}{4\pi\epsilon} \left( \frac{1}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+d)^2}} \right) \quad (5)$$

The electro static potential in area  $z > 0$  is super position of point charge  $+q$  and its image  $-q$ . Once the potential function is obtained, the electric field can be calculated directly from the potential by using equation (6) [12]:

$$E = -\nabla V = \frac{q}{4\pi\epsilon} \left( \frac{\hat{x}x + \hat{y}y + \hat{z}(z-d)}{(x^2+y^2+(z-d)^2)^{3/2}} - \frac{\hat{x}x + \hat{y}y + \hat{z}(z+d)}{(x^2+y^2+(z+d)^2)^{3/2}} \right) \quad (6)$$

**C. Two Dimensional Charge Field**

Simulation of two-dimensional charge field can be calculated with some line charges depicted on the x and y axis, with coefficients of line charge can be written by equation (7)[13]:

$$P_n = \frac{1}{2\pi\epsilon} \ln \sqrt{\frac{(x-x_n)^2 + (y-y_n)^2}{(x-x_n)^2 + (y+y_n)^2}} \quad (7)$$

Where  $\epsilon$  is permittivity,  $(x_n, y_n)$  is coordinate online charge which forms two-dimensional plane, and  $(x, y)$  is the coordinate of measurement point. In the Gauss theorem explained that the line charge located at  $y = 0$  (ground), the potential is zero. By adjusting the boundary conditions on the components of the  $x$  and  $y$  coordinates that form a two-dimensional plane, then the electric field at any point due to  $n$  charges that form two-dimensional plane can be calculated by the following equation:

For  $n$  conductor, the potential of each is [11]:

$$V_1 = \frac{\rho_1}{2\pi\epsilon} \ln \frac{L_{11}'}{L_{11}} + \frac{\rho_2}{2\pi\epsilon} \ln \frac{L_{12}'}{L_{12}} + \frac{\rho_3}{2\pi\epsilon} \ln \frac{L_{13}'}{L_{13}} + \dots + \frac{\rho_n}{2\pi\epsilon} \ln \frac{L_{1n}'}{L_{1n}} \quad (8)$$

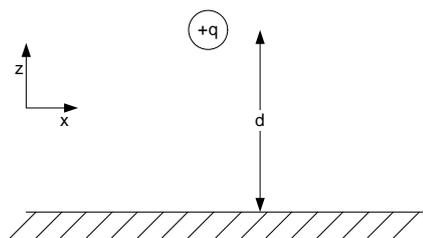


Figure 3. A point charge +q near infinite conductor

$$V_2 = \frac{\rho_1}{2\pi\epsilon} \ln \frac{L_{21}'}{L_{21}} + \frac{\rho_2}{2\pi\epsilon} \ln \frac{L_{22}'}{L_{22}} + \frac{\rho_3}{2\pi\epsilon} \ln \frac{L_{23}'}{L_{23}} + \dots + \frac{\rho_n}{2\pi\epsilon} \ln \frac{L_{2n}'}{L_{2n}} \quad (9)$$

$$V_3 = \frac{\rho_1}{2\pi\epsilon} \ln \frac{L_{31}'}{L_{31}} + \frac{\rho_2}{2\pi\epsilon} \ln \frac{L_{32}'}{L_{32}} + \frac{\rho_3}{2\pi\epsilon} \ln \frac{L_{33}'}{L_{33}} + \dots + \frac{\rho_n}{2\pi\epsilon} \ln \frac{L_{3n}'}{L_{3n}} \quad (10)$$

$$V_n = \frac{\rho_1}{2\pi\epsilon} \ln \frac{L_{n1}'}{L_{n1}} + \frac{\rho_2}{2\pi\epsilon} \ln \frac{L_{n2}'}{L_{n2}} + \frac{\rho_3}{2\pi\epsilon} \ln \frac{L_{n3}'}{L_{n3}} + \dots + \frac{\rho_n}{2\pi\epsilon} \ln \frac{L_{nn}'}{L_{nn}} \quad (11)$$

The above equation can be written in matrix format follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \dots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \dots & P_{2n} \\ P_{31} & P_{32} & P_{33} & \dots & P_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \dots & P_{nn} \end{bmatrix} \cdot \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_n \end{bmatrix} \quad (12)$$

or

$$[V] = [P] \cdot [\rho] \quad (13)$$

Thus, the line charge can be found using the equation:

$$[\rho] = [P]^{-1} \cdot [V] \quad (14)$$

where:

V =potential phase

P =matrix coefficients of Maxwell

$\rho$  =line charge

#### D. Application of (CSM) for the Computation of the Electric Field in Power Transformer

For three-phase power transformer, where the number conductors more than one (R, S, T on the high voltage side and the R, S, T on the low voltage side), the magnitude of the potential at a point is the number of potential caused by their conductor on each side of the high voltage 150 kV and low voltage 20 kV potential transformer windings shown in Figure 5.

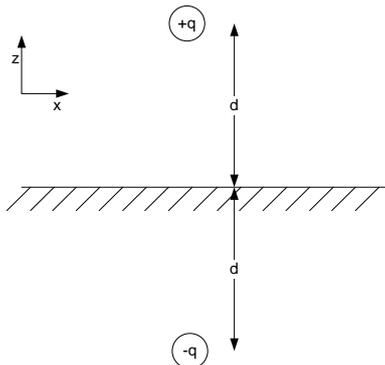


Figure 4. A point charge +q and its image -q

### III. METHODOLOGY

#### A. Specification of Measurement

The specifications of power transformer that used in simulation are:

- Voltage ( $V_{1(\text{rms})}$ ) : 150,000 V (150kV)
- Voltage ( $V_{2(\text{rms})}$ ) : 20,000 V (20kV)
- Transformer dimension: 2.5 x 1.5 x 1 m
- Conductor distance to  $V_1$ :1.25 m
- Conductor distance to  $V_2$ :0.5 m

The Software that used in the simulation is Matlab 2007. Electric field strength is expressed in the vertical axis and horizontal axis, each with real and imaginary parts, or with magnitude and phase angle.

- Vertical axis of electric field strength:

$$E_y = E_{ry} + jE_{iy} \text{ or } E_y = |E_y| < \phi_{ty}$$

- Vertical axis of electric field strength:

$$E_x = E_{rx} + jE_{ix} \text{ or } E_x = |E_x| < \phi_{tx}$$

Voltage equations of each phase are:

$$V_{R1} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos 0 + j \sin 0]$$

$$V_{S1} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos -120 + j \sin -120]$$

$$V_{T1} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos 120 + j \sin 120]$$

$$V_{R2} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos 0 + j \sin 0]$$

$$V_{S2} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos -120 + j \sin -120]$$

$$V_{T2} = \frac{V_{01}}{\sqrt{3}} \sqrt{2} [\cos 120 + j \sin 120]$$

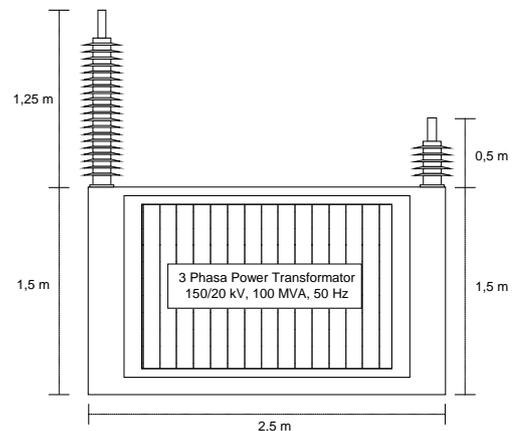


Figure 5. Front view of 3 phasa power transformer, 100 MVA 150 kV/20 kV

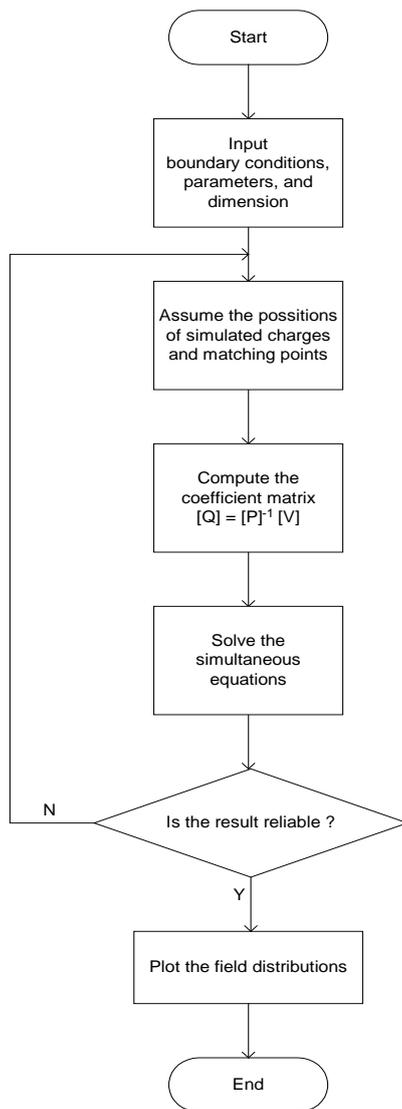


Figure 6. Flow chart of CSM

**B. Boundary Conditions**

The boundary conditions are determined by adjusting the position of a point charge (discretization) with layout coordinates (x,y) on the boundary dimensions of the system to be measured with the following:

- Measurement distance (h) : 5 m
- Radius of phase conductor : 0.007727 m
- Permittivity of air (ε<sub>0</sub>) : 1
- Permittivity of insulation (ε<sub>r</sub>) : 4.5

The difference of permittivity will affect the magnitude of discrete charges around the area of measurement. Since the permittivity of air (ε<sub>0</sub>) is less than the relative permittivity of the insulator (ε<sub>r</sub>), the breakdown voltage will be greater. So that the electric field around the wire conductor will be larger than the electric field around the transformer. Because there is some n charge in the system, then all the electric field of the system is calculated by equation (11).

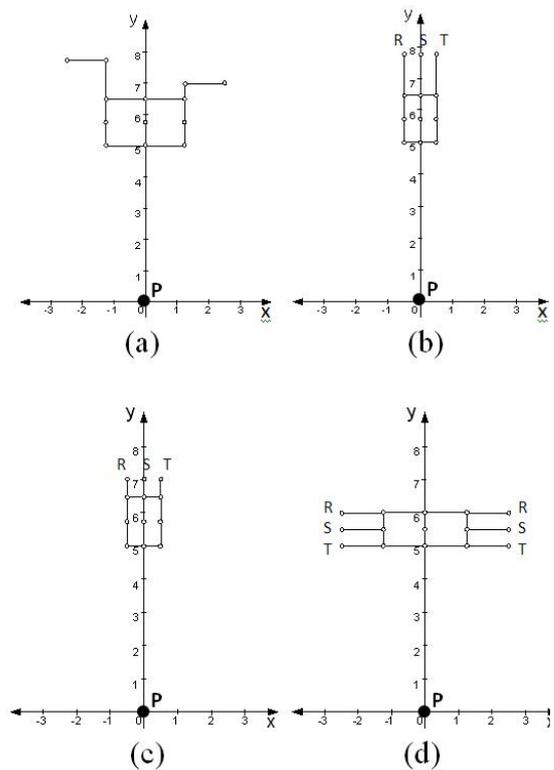


Figure 7. Coordinates of fictitious charge (a) front view, (b) 150 kV-side, (c) 20 kV-side, (d) above view

Figure 6 is a flow chart, and the results of charge discretization on transformer will be the calculation of the electric field from all sides. From Figure 7 can be explained as follows:

- a. Coordinates (x, y) of front view:  
 $x = (-1.25, 0, 1.25, -1.25, 0, 1.25, -1.25, 0, 1.25, 1.25, 2.5, 0, -2.5, -1.25)$   
 $y = (5, 5, 5, 5.75, 5.75, 5.75, 6.5, 6.5, 6.5, 7, 7, 7.35, 7.75, 7.75)$
- b. Coordinates (x, y) of 150 kV side:  
 $x = (-0.5, 0, 0.5, -0.5, 0, 0.5, -0.5, 0, 0.5, -0.5, 0, 0.5)$   
 $y = (5, 5, 5, 5.75, 5.75, 5.75, 6.5, 6.5, 6.5, 7.75, 7.75, 7.75)$
- c. Coordinates (x, y) of 20 kV side:  
 $x = (-0.5, 0, 0.5, -0.5, 0, 0.5, -0.5, 0, 0.5, -0.5, 0, 0.5)$   
 $y = (5, 5, 5, 5.75, 5.75, 5.75, 6.5, 6.5, 6.5, 7, 7)$
- d. Coordinates (x, y) of above view:  
 $x = (-2.5, -1.25, 0, 1.25, 2.5, -2.5, -1.25, 0, 1.25, 2.5, -2.5, -1.25, 0, 1.25, 2.5)$   
 $y = (5, 5, 5, 5, 5, 5.5, 5.5, 5.5, 5.5, 5.5, 6, 6, 6, 6, 6)$

Coordinates of measurement point for all view measurement located at the point P (0,0).

#### IV. RESULTS AND DISCUSSIONS

The simulation has been done using the Mat lab, the distribution of the electric field and equipotential lines from each side of the transformer three-phase 100 MVA, 150/20 kV at a distance of 5 meters measurements are shown in Figure 8 and Figure 9.

Electric field distribution shown is the electric field strength, without indicating its direction, which is the vector sum of real and imaginary electric field in the direction x (horizontal) and y (vertical).

The electric field generated from the front view of transformer has a maximum value at a distance of -2.6 m from the x-axis measurement point, with magnitude of electric field strength 6833 V/m, respectively. While at the measurement point (5 m), the resulting electric field is higher than WHO standard, amounting to 5541 V/m. According to equation (1), the farther the distance from the maximum value ( $x < -2.6$  m and  $x > -2.6$  m), the smaller the distribution of electric field is.

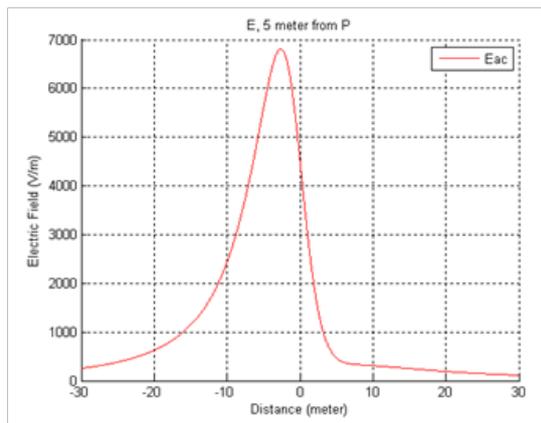


Figure 8. Simulated and measured electric field in front view of power transformer

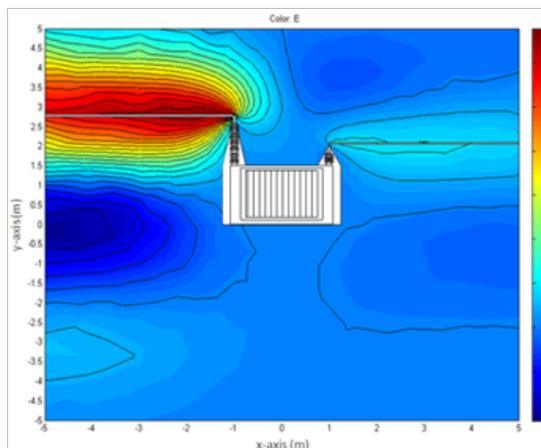


Figure 9. Equipotential distribution in front view of power transformer

The electric field generated from 150 kV-side of transformer has a maximum value at a distance of -3 m and 3 m from the x-axis measurement point, with magnitude of electric field strength, 3050 V/m (shown in Figure 10 and Figure 11). While at the measurement point, the resulting electric field is smaller than WHO standard, amounting to 1930 V/m. The farther the distance from the maximum value ( $x < -3$  m and  $x > 3$  m), the smaller the distribution of electric field is.

The electric field generated from 20 kV-side of transformer has a maximum value at a distance of -3 m and 3 m from the x-axis measurement point, with magnitude of electric field strength 502 V/m, respectively (shown in Figure 12 and Figure 13). Mean while, at the measurement point (5 m), the resulted electric field is smaller at the amount of 427 V/m. The farther the distance from the maximum value ( $x < -3$  m and  $x > 3$  m), the smaller the distribution of electric field.

The electric field generated from above of transformer has a maximum value at a distance

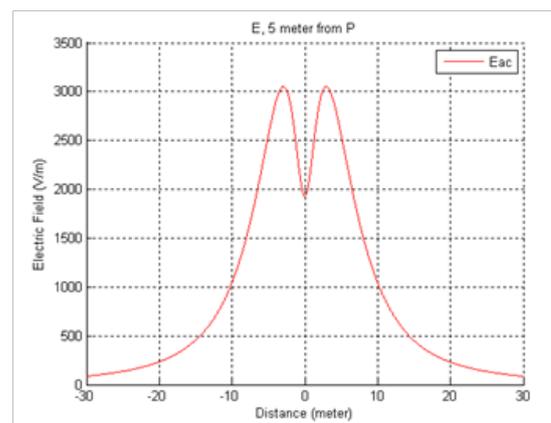


Figure 10. Simulated and measured electric field of 150kV-side

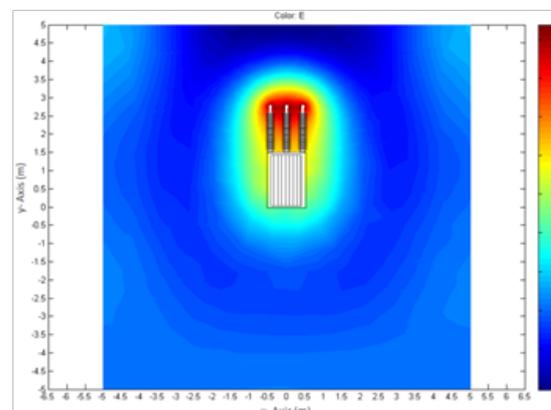


Figure 11. Equipotential distribution of 150 kV-side

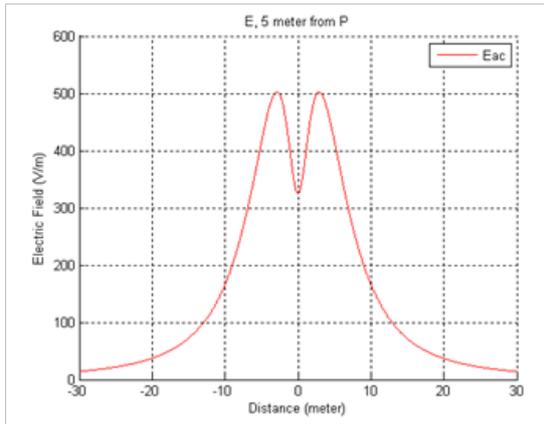


Figure 12. Simulated and measured electric field of 20kV-side

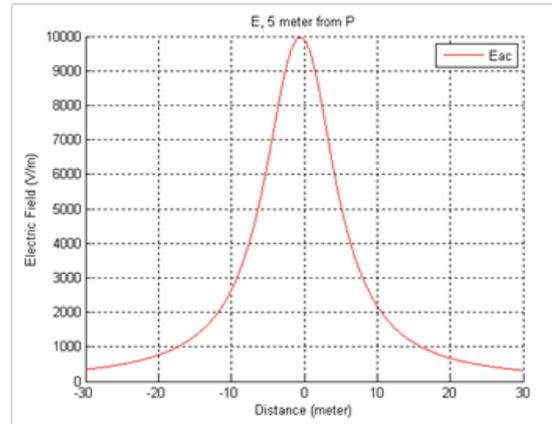


Figure 14. Simulated and measured electric field from above of power transformer

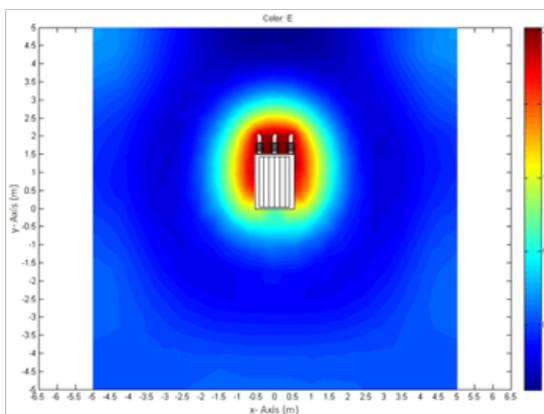


Figure 13. Equipotential distribution of 20kV-side

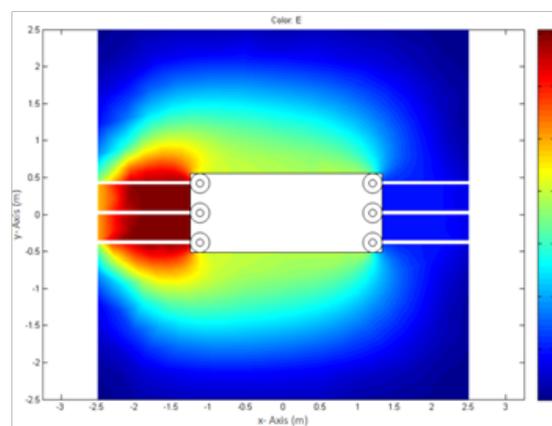


Figure 15. Equipotential distribution from above of power transformer

of -0.5 m from the x-axis measurement point, with magnitude of electric field strength 9960 V/m, respectively (shown in Figure 14 and Figure 15). While at the measurement point, the resulting electric field is smaller, in the amount of 9890 V/m. The farther the distance from the maximum value ( $x < -0.5$  m and  $x > 0.5$  m), the smaller the distribution of electric field is.

From all measurement results, the electric field around the wire conductor (R, S, T) without isolation is greater than the electric field around an isolated transformer wall, this is due to differences in the dielectric permittivity ( $\epsilon$ ) of a medium that causes the breakdown voltage differences that affect the magnitude of the electric field generated.

The level of precision and measurement error in the simulation results depend on the determination of the location of the position discretization contour points to be measured as well as the boundary conditions as an important parameter in measuring and mapping the electric field.

## V. CONCLUSION

From the simulation results of the electric field in the area of the transformer, it can be concluded that simulation results shows the magnitude of the electric field generated in power transformer at the measurement point (5 m) is 5541 V/m. This can be considered that the safety limit of the electric field effect is in accordance with the WHO (World Health Organization), which is 5 kV/m. Moreover, CSM is efficient for calculating the electric field with fairly simple programs and less computing time. The surface of the electrodes and the polarization charges on the interface of different dielectrics are replaced by a set of discrete simulated charge. The types and positions of the simulated charges are predetermined. The magnitudes of these equivalent charges are determined by the boundary conditions on the collocation points of the boundary. Hence, CSM is one of the collocation methods and can be classified as an equivalent source method.

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